Announcements:

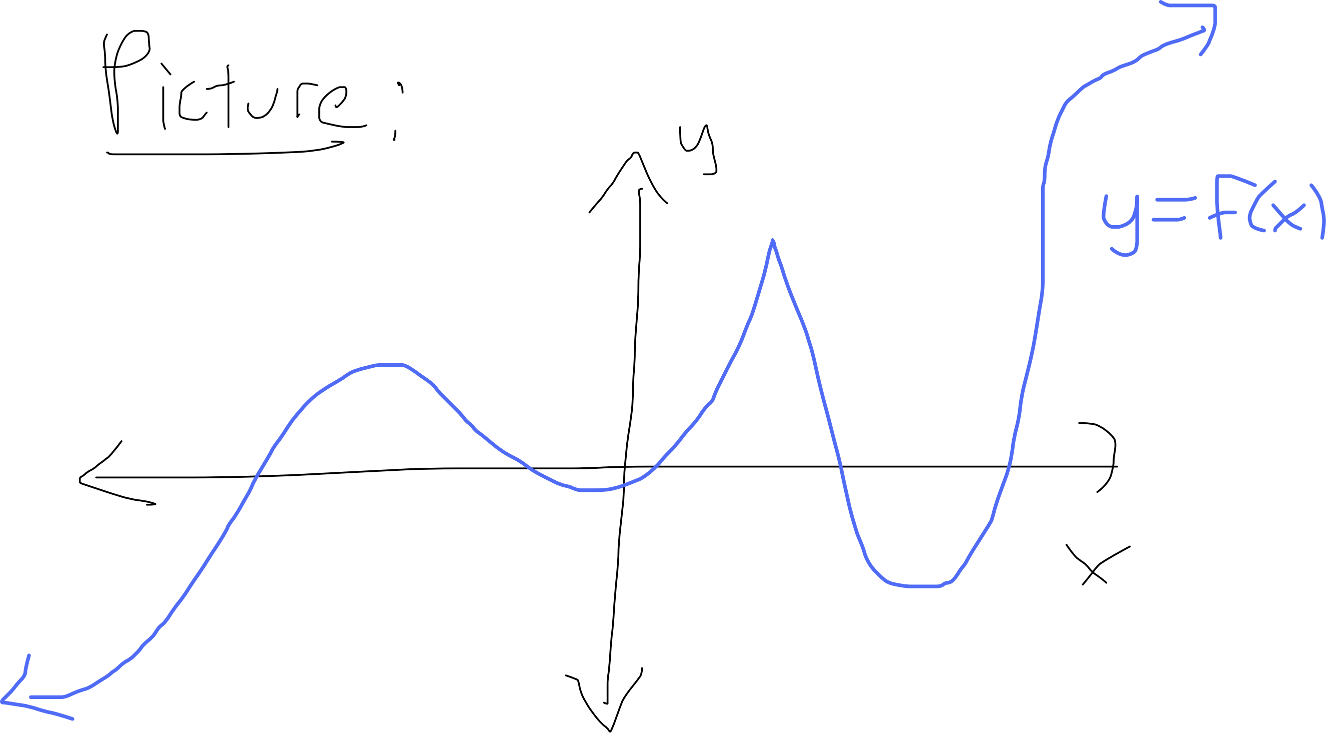
1) Exam I moved to Thursday next week.

2) HW due later tonight.

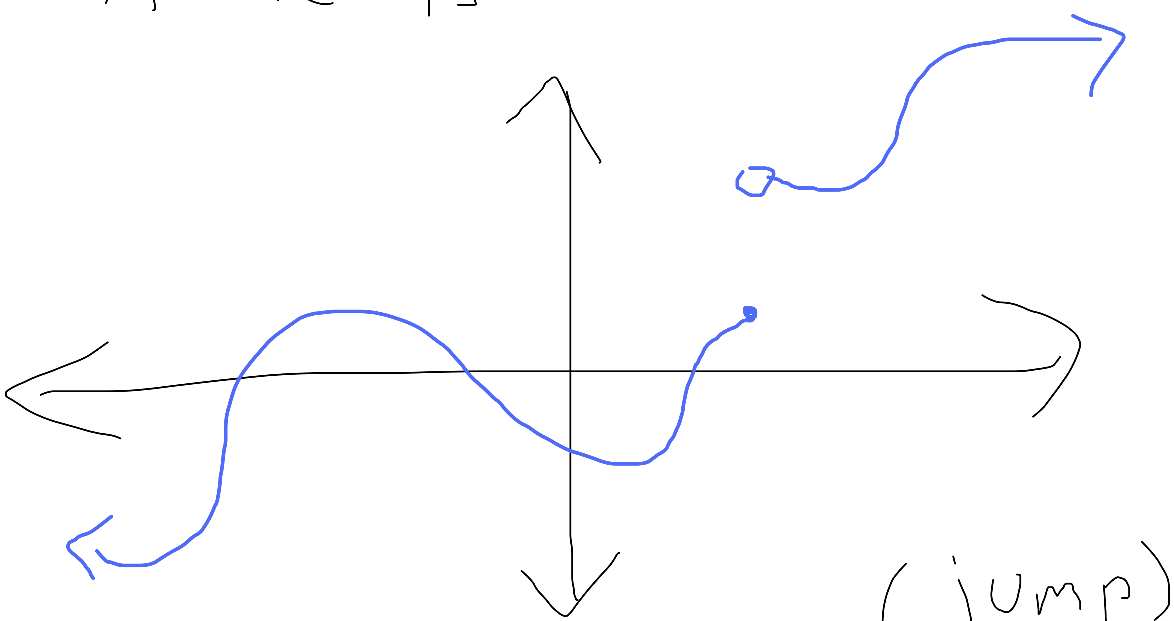
Continuity (section 1.8)

The idea: f is continuous for all real numbers if you can draw the graph of f without picking up your pencil from paper.

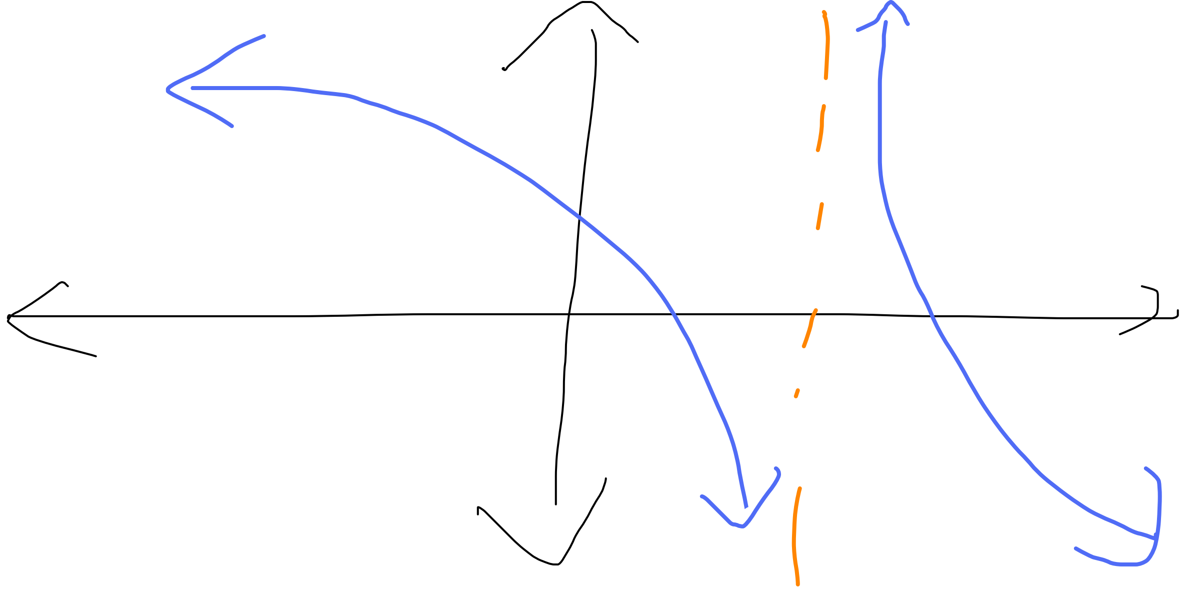
Picture:



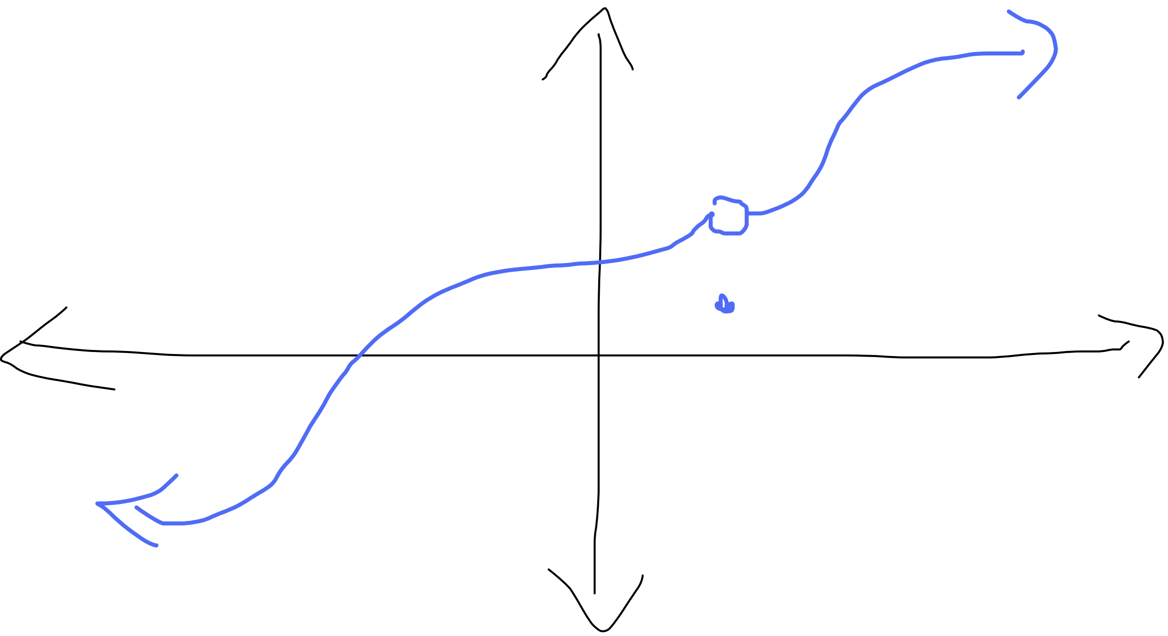
Above is a continuous function.



Discontinuous Function



Discontinuous (infinite)



Discontinuous (removable)

Temperature is usually regarded as a continuous function of time.

Time itself can be thought of as continuous - but as a function of what?

Formal Definition: (left, right, interval)

f is continuous from the left at $x = a$ if

$$\boxed{\lim_{x \to a^-} f(x) = f(a)}$$

f is continuous from the right at $x = a$ if

$$\boxed{\lim_{x \to a^+} f(x) = f(a)}$$

f is continuous at $x = a$

if $\boxed{\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)}$

We say f is continuous if

it is continuous at each point

in its domain.

We say f is continuous on $[a, b]$

if f is continuous on (a, b),

left - continuous at $x = b$, and

right continuous at $x = a$

Example 1. Is the function

$$f(x) = \begin{cases} 7\sin(x), & x > \frac{\pi}{3} \\ \frac{7}{2}, & x = \frac{\pi}{3} \\ x^2 - \frac{2\pi}{3}x + \frac{\pi^2}{9} + \frac{7}{2}, & x < \frac{\pi}{3} \end{cases}$$

continuous at $x = \frac{\pi}{3}$.

Show Whether

$$\lim_{x \to \frac{\pi}{3}^-} f(x) = f\left(\frac{\pi}{3}\right) = \lim_{x \to \frac{\pi}{3}^+} f(x).$$

$$f\left(\frac{\pi}{3}\right) = \boxed{\frac{7}{2}}$$

$$\lim_{x \to \frac{\pi}{3}^-} f(x)$$

$$= \lim_{x \to \frac{\pi}{3}^-} \left(x^2 - \frac{2\pi}{3}x + \frac{\pi^2}{9} + \frac{7}{2}\right)$$

$$= \frac{\pi^2}{9} - \frac{2\pi}{3} \cdot \frac{\pi}{3} + \frac{\pi^2}{9} + \frac{7}{2}$$

$$= \cancel{\frac{2\pi^2}{9}} - \cancel{\frac{2\pi^2}{9}} + \frac{7}{2}$$

$$= \boxed{\frac{7}{2}}$$

$$\lim_{x \to \frac{\pi}{3}^+} f(x) = \lim_{x \to \frac{\pi}{3}^+} 7\sin(x)$$

$$= 7\sin\left(\frac{\pi}{3}\right)$$

$$= 7 \cdot \frac{\sqrt{3}}{2}$$

$$\neq \frac{7}{2}$$

So f is not continuous at $x = \frac{\pi}{3}$.

Example 2:

Find where

$$f(x) = \begin{cases} x^2 - 3x, & x \geq 2 \\ x - 4, & -1 < x < 2 \\ \cos(\pi x), & -3 \leq x \leq -1 \\ -1, & x < -3 \end{cases}$$

is continuous.

Potentially discontinuous at only $x = 2, -1,$ and -3.

$\underline{x=2}$

$$f(2) = 2^2 - 3 \cdot 2 = \boxed{-2}$$

$$\left(f(x) = x^2 - 3x \text{ at } x = 2 \right)$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(x^2 - 3x \right) = \boxed{-2}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(x - 4 \right) = \boxed{-2}$$

f is continuous at $x = 2$.

$\underline{x=-1}$ $\quad f(-1) = \cos(-\pi) = \boxed{-1}$

$$\lim_{x \to -1^+} f(x) = -1-4 = -5$$

$$\neq -1$$

(we use $x-4$ instead of x^2-3x since $x-4$ is closer to -1 in the definition)

So f is not continuous at $x=-1$.

$\underline{x = -3}$ $\quad f(-3) = \cos(-3\pi) = \boxed{-1}$

$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} \cos(\pi x)$$

$$= \cos(-3\pi) = \boxed{-1}$$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} -1 = \boxed{-1}$$

Conclusion: f is continuous at $\boxed{x = -1}$

Answer: f is continuous

on $\boxed{(-\infty, -1) \cup (-1, \infty)}$

("$\cup$" means "or")

Could also write

$$\boxed{x \neq -1}$$